

# ***The Thought of Combining Numbers and Shapes: A Bridge of Thinking Connection Between High School Mathematics and College Mathematics***

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**Abstract.** The thought of combining numbers and shapes is a core ideological method running through the development of mathematics, with "solving shapes with numbers" and "aiding numbers with shapes" as its core connotations, and it adheres to the principles of equivalence, bidirectionality and simplicity. From the ancient Pythagorean Theorem to the establishment of modern analytic geometry, it has achieved systematic theoretical formation. High school mathematics uses the thought of combining numbers and shapes as a tool to solve concrete mathematical problems, while college mathematics sublimates it into a core method for theoretical research and logical deduction. The two are not isolated but progressive and deeply connected. The thought of combining numbers and shapes is not only the key to solving problems in high school mathematics, but also the thinking foundation for learning college mathematics and applied mathematics majors. Starting from the basic cognition of the thought of combining numbers and shapes, this paper analyzes its typical applications in high school mathematics, focuses on exploring the connection relationship of this thought in high school and college mathematics learning, and reveals its important value for mathematics major learning. This paper can provide reference for the connection of mathematics teaching and the cultivation of students' thinking.

**Keywords:** combining numbers and shapes, high school mathematics, college mathematics, analytic geometry, mathematical thought

## **1. Introduction**

Mathematics is the science that studies quantitative relations and spatial forms. As the two core research objects of mathematics, the organic combination of numbers and shapes constitutes the foundation of the mathematics discipline [1]. The thought of combining numbers and shapes transforms abstract algebraic language and intuitive geometric figures into each other, realizing the unity of abstract thinking and image thinking [2]. In the high school mathematics teaching system, combining numbers and shapes is the core method to solve problems such as functions, analytic geometry and inequalities, and it is also a key ability examined in the college entrance examination mathematics [3]. For college students majoring in mathematics and applied mathematics, the thinking of combining numbers and shapes formed in high school is the key to understanding

professional courses such as advanced mathematics, mathematical analysis, analytic geometry and differential geometry [4].

From the perspective of the knowledge system, high school mathematics is the preparatory stage of college mathematics, and college mathematics is the deepening and expansion of high school mathematics [5]. As an important link connecting the two, the thought of combining numbers and shapes not only realizes the connection at the knowledge level, but also completes the advancement at the thinking level [6]. Based on the basic cognition and typical high school applications of the thought of combining numbers and shapes, this paper systematically analyzes its relationship in high school and college mathematics learning, and provides theoretical reference for mathematics major students to sort out the thinking context and improve learning efficiency.

## **2. Basic cognition of the thought of combining numbers and shapes**

The thought of combining numbers and shapes is one of the oldest and most core ideological methods in mathematics. Its connotation, principles and historical evolution have constructed a complete theoretical system, which is the basis for understanding the connection between high school and college mathematics [7].

### **2.1. Core connotation: the dialectical unity of "solving shapes with numbers" and "aiding numbers with shapes"**

The core of the thought of combining numbers and shapes is to realize the mutual transformation of numbers and shapes, which includes two dimensions: "aiding numbers with shapes" and "solving shapes with numbers". The two are opposites yet unified, complementing each other. "Aiding numbers with shapes" refers to transforming abstract algebraic problems into concrete geometric problems with the help of the intuitiveness of geometric figures, reducing the difficulty of thinking [8]. For example, using function images to judge the monotonicity of functions and solve the solution sets of inequalities, representing the magnitude relationship of real numbers through the number axis, and presenting algebraic laws intuitively with figures. This method conforms to the thinking characteristics of high school students and can quickly sort out problem-solving ideas [9].

"Solving shapes with numbers" accurately depicts the essential characteristics of geometric figures through algebraic operations and equation derivation, making up for the limitations of geometric intuition [10]. For instance, using the distance formula to judge the positional relationship between a line and a circle, solving conic section parameters through coordinate operations, and strictly proving geometric theorems with algebraic methods to avoid errors in intuitive judgment. This dimension focuses on logical reasoning and accurate calculation, which is the core thinking mode of college mathematics research [11].

The two are dialectically unified, realizing the integration of abstract thinking and image thinking. High school mathematics focuses on the practical problem-solving of "aiding numbers with shapes", while college mathematics strengthens the rigorous deduction of "solving shapes with numbers", which together constitute the complete connotation of the thought of combining numbers and shapes [12].

### **2.2. Key principles: equivalence, bidirectionality and simplicity**

The application of the thought of combining numbers and shapes needs to follow three core principles, which are also the common criteria for high school problem-solving and college research

[13]. The equivalence principle is the core bottom line, requiring that the transformation of numbers and shapes maintains logical equivalence, and algebraic relations correspond to geometric features one by one to avoid deviation in the transformation scope. This is the basis for ensuring mathematical rigor and the key to avoiding mistakes in high school problem-solving [14].

The bidirectionality principle emphasizes that the transformation of numbers and shapes is reversible, which can be transformed from algebra to geometry and also from geometry back to algebra. This principle breaks the limitation of single thinking. In high school, it is used to optimize problem-solving methods, and in college, it is relied on to construct a theoretical system. The corresponding relationship between curves and equations in analytic geometry is a typical embodiment [15].

The simplicity principle is the ultimate goal of applying the combination of numbers and shapes. It simplifies problems and reduces difficulty through the transformation of numbers and shapes, which is always followed whether simplifying operation steps in high school or simplifying theoretical derivation in college [16].

### **2.3. Historical context: the evolution of thought from the Pythagorean Theorem to Descartes' analytic geometry**

The thought of combining numbers and shapes has a long history and is a microcosm of the progress of human mathematical thinking [17]. In ancient mathematics, the combination of numbers and shapes was in the stage of intuitive application. The Chinese Pythagorean Theorem realized the preliminary combination of numbers and shapes through the side length relationship of right triangles, and ancient Greek mathematicians also studied quantitative relations based on geometric figures, laying the foundation for classical combination of numbers and shapes [18].

In modern times, Descartes founded analytic geometry, introduced the plane rectangular coordinate system, established the corresponding relationship between points and real number pairs, curves and equations, and achieved a revolutionary breakthrough in the thought of combining numbers and shapes. It marked that this thought was sublimated from scattered skills to a systematic theory, and also laid the foundation for the establishment of calculus [19]. This evolution process is highly consistent with the mathematics learning path: high school corresponds to the intuitive application stage, and college focuses on the theoretical deepening stage [20].

## **3. Typical embodiments of the thought of combining numbers and shapes in high school mathematics**

High school mathematics centers on concrete problems, and the thought of combining numbers and shapes is widely used in the two core modules of analytic geometry and functions. It is not only the key to solving problems, but also the pre-pavement for college mathematics knowledge.

### **3.1. Application in analytic geometry**

Analytic geometry is the most direct carrier of the thought of combining numbers and shapes. It builds a bridge between numbers and shapes through the coordinate system, and its core applications are concentrated in the two modules of lines and circles, and conic sections.

### 3.1.1. Positional relationship between a line and a circle

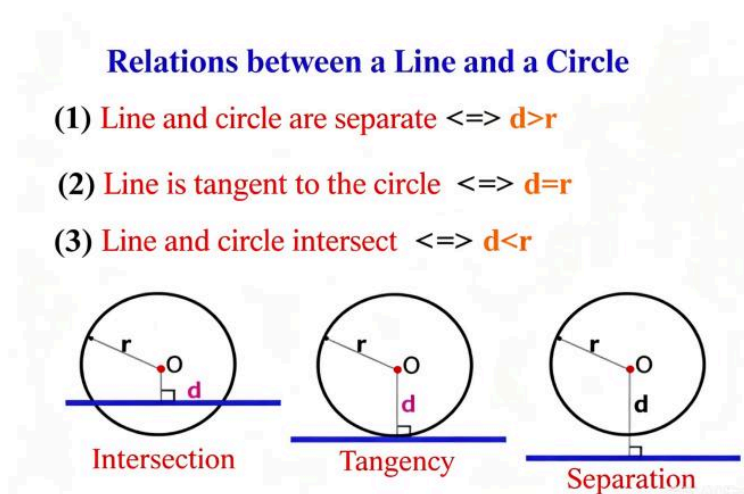


Figure 1. Three positional relationships between a line and a circle ( $d$  is the distance from the center of the circle to the line,  $r$  is the radius)

The positional relationship between a line and a circle is a basic question type in high school analytic geometry, which perfectly integrates the logic of combining numbers and shapes [21]. At the geometric level, the relationships of intersection, tangency and separation can be judged through intuitive observation; at the algebraic level, accurate judgment can be realized by comparing the distance from the center of the circle to the line with the radius. High school problem-solving often adopts the mode of "graph prediction + algebraic verification", which not only cultivates geometric intuition ability, but also strengthens operation ability. This content is also the basis of the college course *Analytic Geometry*. In college, it is extended to three-dimensional space geometry to study the positional relationships between lines and planes, and between planes, with the core methods being in the same line [22].

### 3.1.2. Properties of conic sections

Conic sections are the in-depth application of the thought of combining numbers and shapes in high school. The geometric definitions and standard equations of ellipses, hyperbolas and parabolas are highly unified. In high school, students need to flexibly realize the mutual transformation of definitions, figures and equations, simplify operations with the help of geometric properties, and prove geometric features through equation derivation. This part of knowledge is the pre-foundation of college *Differential Geometry* and *Higher Geometry*. College courses extend it to the category of projective geometry, realizing the leap from concrete research to theoretical research.

### 3.2. Application in functions

Functions are the main line of high school mathematics, and the combination of numbers and shapes runs through the whole process of function learning. Visualizing abstract function properties through images is the core method to solve function problems.

### 3.2.1. Analysis of function properties

It is relatively abstract to understand the core properties of functions such as monotonicity and parity solely relying on algebraic definitions, but it is intuitive and easy to understand when combined with images. The image of an odd function is symmetric about the origin, and that of an even function is symmetric about the y-axis. The monotonicity can be directly judged by the rising and falling trend of the image. With this method, problems such as inequalities and maximum values can be solved quickly. This thinking of "intuition first, rigorous verification" is an important foundation for college mathematical analysis learning. In college, function properties are extended to uniform continuity, concavity and convexity, and images are still needed to establish intuitive cognition.

### 3.2.2. Judgment of equation roots and function zeros

Function zeros are equivalent to equation roots. For high-order and transcendental equations, algebraic solution is difficult, and the combination of numbers and shapes provides an efficient solution method. By drawing function images and observing the number and position of their intersection points with the x-axis, the distribution of equation roots can be judged. This method is also the core idea of college *Numerical Analysis* and *Mathematical Modeling*. On this basis, colleges have formed a systematic approximate solution method, whose logical core is completely consistent with that of high school.

## 4. The relationship between high school mathematics and college mathematics from the perspective of combining numbers and shapes

### 4.1. Knowledge level: progressive connection and consistent inheritance

The knowledge systems of high school mathematics and college mathematics are not isolated, but form a progressive connection with the combination of numbers and shapes as the link. High school analytic geometry is the foundation of college space analytic geometry, high school functions are the pre-content of college mathematical analysis and complex functions, and high school problem-solving skills of combining numbers and shapes are the basic tools for college mathematics research. High school focuses on the skill application of specific question types, while college focuses on the construction and deepening of the theoretical system. The difficulty of knowledge gradually increases, but the core thought is always unified.

### 4.2. Thinking level: from intuitive concreteness to abstract rigor

The combination of numbers and shapes in high school focuses on intuitive problem-solving, aiming at simplifying problems and obtaining correct results; the combination of numbers and shapes in college focuses on rigorous deduction, with logical strictness and theoretical proof as the core. The geometric intuition ability cultivated in high school is the foundation of college learning, and college requires completing strict logical proof on the basis of intuition to realize the advancement of thinking ability. The thought of combining numbers and shapes helps students complete the thinking transformation smoothly and adapt to the abstract reasoning requirements of college mathematics.

### 4.3. Ability level: from problem-solving skills to research methods

In high school, combining numbers and shapes is a problem-solving skill for exams; in college, it is sublimated into a core method for mathematical theoretical research and practical problem solving. The core literacy of mathematics majors is to use mathematical thoughts to solve problems. Whether it is mathematical modeling, numerical calculation, or differential equation solving, it is inseparable from the combination of numbers and shapes. The ability of combining numbers and shapes accumulated in high school is the starting point for the cultivation of college professional abilities.

## 5. Conclusion

As the core thought of mathematics, the thought of combining numbers and shapes has built a bridge connecting high school mathematics and college mathematics. Its dialectical connotation of "solving shapes with numbers and aiding numbers with shapes", three application principles and complete historical evolution provide theoretical guidance for mathematics learning. In high school, combining numbers and shapes is the core of solving problems in analytic geometry and function modules; in college, it becomes a research method supporting professional courses such as advanced mathematics and mathematical analysis.

Combining numbers and shapes is not only a problem-solving tool, but also a core literacy running through mathematics learning. For students majoring in mathematics and applied mathematics, the thought of combining numbers and shapes is the key to realizing the transformation from concrete problem-solving in high school to abstract research in college. Only by firmly mastering the application foundation of combining numbers and shapes in high school and deeply understanding its ideological connotation can students smoothly adapt to the rhythm of college mathematics learning and improve their professional mathematics literacy. In the whole process of mathematics learning, we should always implement the thought of combining numbers and shapes, realize the organic unity of numbers and shapes, and promote the continuous advancement of mathematical thinking. Adhering to the thought of combining numbers and shapes is conducive to realizing the smooth transition and continuous improvement of mathematical thinking from high school to college.

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